Equations of Tangents & Normals (Implicit)

ex) Find the equation of the tangent line to $x^2+y^2=4$ at $(1,\sqrt{3})$.

Need $\frac{dy}{dx}$ (slope of tangent)

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-2x}{2y}$$

$$\frac{dy}{dx}=\frac{-x}{y}$$

$$\left.\frac{dy}{dx}\right|_{(1,\sqrt{3})}=\frac{-1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{-\sqrt{3}}{3}$$

$$y=mx+b$$

$$(\sqrt{3})=-\frac{\sqrt{3}}{3}(1)+b$$

$$\frac{3\sqrt{3}}{3}=-\frac{\sqrt{3}}{3}+b$$

$$b=4\sqrt{3}/3$$

$$y=-\frac{\sqrt{3}}{3}x+\frac{4\sqrt{3}}{3}$$

ex) Find the equation of the tangent line to $x^3+y^3=\underbrace{6x}\underbrace{y}$ at $(3,3)$.

$$3x^2+3y^2\frac{dy}{dx}=6y+6x\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx}-6x\frac{dy}{dx}=6y-3x^2$$

$$\frac{dy}{dx}=\frac{6y-3x^2}{3y^2-6x}=\frac{\cancel{3}(2y-x^2)}{\cancel{3}(y^2-2x)}=\frac{2(3)-3^2}{3^2-2(3)}$$

$$=\frac{-3}{3}$$

$$=-1$$

$$y-3=-1(x-3)$$

$$\underline{y=-1x+6}$$

pg. 155-156 #27-32, 48, 49

ex) Find the equation of the normal line to

$$2x^5 + x^4y + y^5 = 36 \quad \text{at } (1,2)$$

$$10x^4 + 4x^3y + x^4\frac{dy}{dx} + 5y^4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(x^4 + 5y^4\right) = -10x^4 - 4x^3y$$

$$\frac{dy}{dx} = \frac{-10x^4 - 4x^3y}{x^4 + 5y^4}$$

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-10(1)^4 - 4(1)^3(2)}{1^4 + 5(2)^4} = \frac{-18}{81}$$

$$= -2/9$$

$$m_{normal} = +9/2$$

$$y - 2 = 9/2(x-1) \quad \ldots \quad y = \frac{9}{2}x - \frac{5}{2}$$

ex) Find $\frac{dy}{dx}$ if $x^2 + \sqrt{y} = x^2 y^3 + 5$.

$$2x + \frac{1}{2} y^{-1/2} \frac{dy}{dx} = 2xy^3 + x^2 3y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{2\sqrt{y}} - x^2 3y^2\right) = 2xy^3 - 2x$$

$$\frac{dy}{dx} = \frac{2x(y^3-1)}{\frac{1}{2\sqrt{y}} - x^2 3y^2} \quad \ldots ?$$